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Switching Surges and Air Insulation

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Switching surges and air insulation

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[Plate 18]

Some thirteen years ago, a reduction was noticed in the strength of air insulation when subjected to slowly rising positive impulse voltages such as occur during switching operatons on power systems. Methods for the prediction and control of switching overvoltages have been established and empirical data collected in high voltage laboratories. Insulation against switching surges is now seen as an important factor in the feasibility of power transmission at ultra-high voltages. The strength of large air gaps depends not only on the geometry of the gap (point–plane, rod–rod, etc.), but also on the waveshape of the applied voltage. The practical diversity of gaps and waveshapes is such that a sound theoretical approach must be found if laboratory testing is to be kept within bounds. Qualitative understanding of the breakdown processes in long air-gaps has advanced rapidly in the last two years. Predictive mathematical models are now being constructed.

1. Introduction

Insulation is stressed by two distinct types of overvoltage; that due to switching operations and that due to lightning. Both types have a wide range of magnitudes, the higher values occurring less frequently. On existing e.h.v. power systems, almost all switching overvoltages are lower than the insulation level of the plant, and the problem is that of balancing the cost of rare insulation failures against the cost of providing more insulation. It has been known for some time that the breakdown voltage of long air gaps when subjected to switching surges is less than proportional to the length of the gap. For the u.h.v. plant presently being considered for transmission at voltages above 1000 kV, it appears that the low switching surge strength of air gaps may be a design limitation. In the last few years, u.h.v. laboratories have been built in several countries and used for the study of breakdown of air clearances representative of electrical transmission plant; the u.h.v. laboratory at Leatherhead is depicted in figure 1, plate 18.

Many attempts have been made to produce probability statistics of switching overvoltages. Whereas overvoltages arising from any given cause do have a meaningful probability distribution, different causes give rise to different mean and maximum values and also to different characteristic waveshapes. Switching overvoltages are commonly described by their peak amplitude, relative to the peak of the power-frequency voltage; this ratio is often termed ‘per unit’ (p.u.). It is important to distinguish two types of overvoltage; those occurring on the main network and those on the item of plant undergoing connexion to or disconnexion from the network. The former are largely governed by the nature of the network and rarely exceed 2.0 p.u., while the latter are governed by the characteristics of the plant and the behaviour of the switch and sometimes exceed 3.0 p.u.

Over the last decade, much work has been done to quantify overvoltages occurring on power systems. Predictive methods, both analogue and digital, have been developed and validated by measurements on the transmission system (Dwek, Hall, Jackson & Jones 1972). The waveshapes of switching overvoltages are often complex and are influenced by many factors. Figure 2 shows two simple examples. The first results from the disconnexion of a shunt inductor by an imperfect switch and is characterized by a frequency of oscillation of a few thousand hertz and an amplitude of between 2 and 3 p.u. The second waveform appears on the open terminals of a transmission line when suddenly energized and is characterized by a step followed by a slow rise and a sharp drop; the duration is commonly 1 to 2 ms and the amplitude 2.0 to 2.5 p.u. Neither of these waveshapes bears much superficial resemblance to the double exponential surge conveniently and conventionally produced in the laboratory by the Marx impulse generator and with which almost all the published data on air gap breakdown have been obtained.

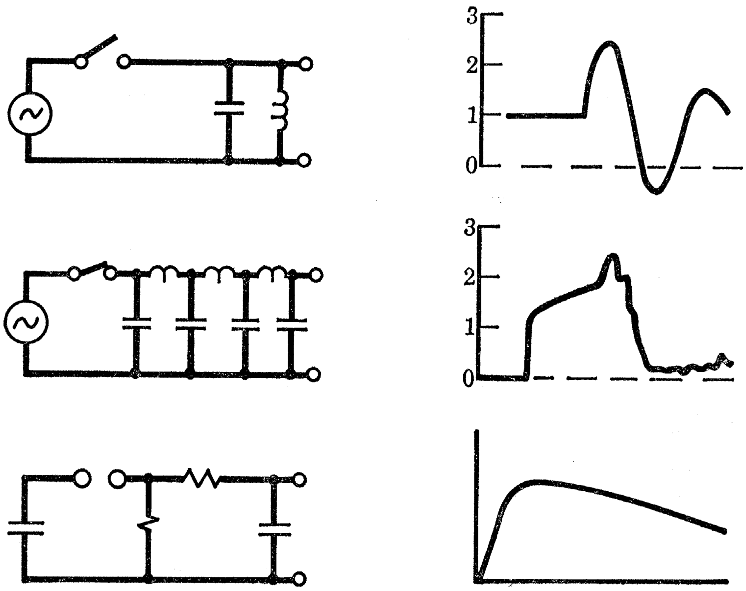


Figure 2. Waveshape of representative switching overvoltage: top, disconnexion of shunt inductance; centre, energization of transmission line; bottom, laboratory switching impulse. Amplitudes are given relative to peak power–frequency voltage.

2. Effect of the voltage waveshape

Stekol'nikov (1961) and Hughes & Roberts (1965) showed that the breakdown voltage of air gaps was sensitive to the time to peak of the applied voltage – the so-called 'U-curve' phenomenon. For any given gap length, the rod–plane geometry was found to give the lowest breakdown voltage. Figure 3 summarizes the positive critical or 50 % flashover voltage of rod–plane gaps from 1 to 10 m, subjected to double exponential surges with times to peak ranging from 2 to 1000 μs (Jones & Whittington 1972*b*). For the shortest wavefronts, the strength of the gap is almost directly proportional to length, corresponding to a mean stress of ~500 kV/m. The time to peak at which the breakdown voltage is a minimum increases in proportion to the length of the gap. The minimum breakdown voltage is roughly proportional to the square root of the gap length and the mean stress is less than 200 kV/m for a 10 m gap.

The mean time to breakdown at the critical flashover voltage is shown in figure 4. For shorter times to peak, the time to breakdown approaches an almost constant value of about 20 μs/m of gap-length. As the time to peak is increased towards the minimum of the voltage U-curve, breakdown tends to occur around the peak of the wave. For longer times to peak, breakdown occurs on the front of the wave, at between 90 and 95 % of the peak voltage. If the applied

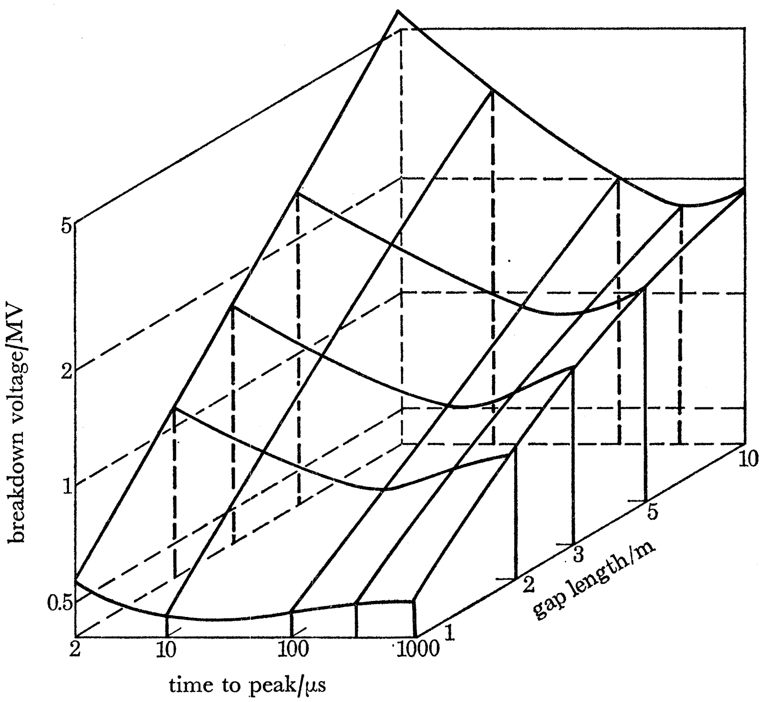


FIGURE 3. Breakdown voltage of rod–plane gaps as a function of gap length and time to peak of applied voltage (Jones & Whittington 1972*b*).

voltage is raised above the critical flashover value, breakdown occurs progressively earlier on the wave at a higher instantaneous voltage. This variation of the time to, and voltage at, breakdown has important consequences for the coordination of insulation, particularly when it is required that an air gap should always break down in preference to the solid insulation of, say, a transformer (Jones 1972).

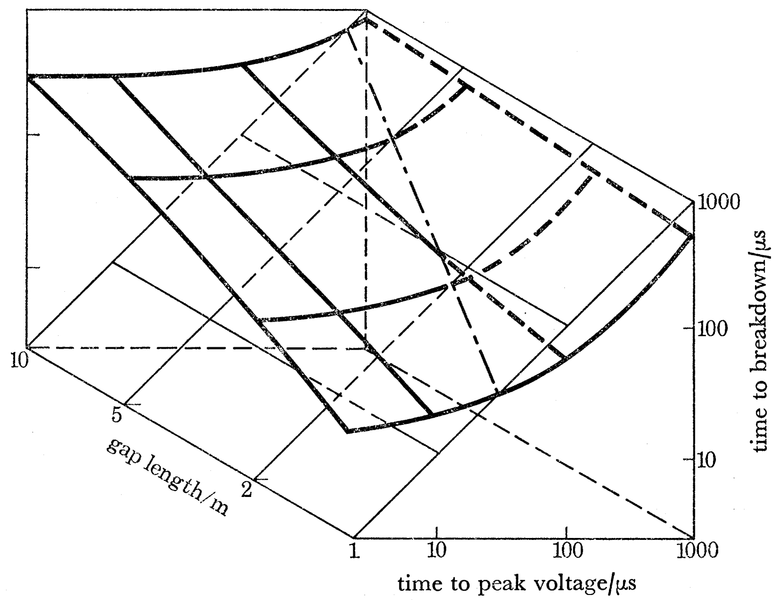


FIGURE 4. Mean time to breakdown of rod–plane gaps as a function of gap length and time to peak of applied voltage (Jones & Whittington 1972*b*).

For the longer air clearances, the increasing sensitivity to time to peak calls into question the validity of laboratory testing with double exponential surges. Jones & Whittington (1972*a*) and Colombo, Sartorio & Taschini (1972) made experiments with oscillatory (1-cosine) waveforms on gaps of 0.8 and 2 m respectively. It was found that the minimum breakdown voltage for the 1-cosine wave was significantly lower than for the double exponential wave and occurred at a longer time to peak; breakdown tended to occur close to the peak of the wave.

3. The effect of electrode geometry

The effect of the geometry of the gap has been explored by Paris & Cortina (1968). A '*K*-factor' for a gap is defined as the ratio of the positive switching surge breakdown voltage to that of a rod–plane gap of the same length. Geometries of engineering importance are assigned a value of *K* (e.g. conductor–plane, 1.25), thus simplifying design estimates. For longer gaps, *K* may no longer be regarded as independent of length or of the wavefront chosen for comparison. A further difficulty in application is that of categorizing certain gaps, e.g. rod–rod, which are influenced by the proximity of earthed surfaces.

An important property of a two-electrode gap is the symmetry of the field, in particular the relative stress concentration at the two electrodes. Even if quite different mechanisms and critical stresses exist for the positive and negative breakdown of rod–plane gaps, provided the field is symmetrical there can be no polarity effect. The positive and negative switching surge breakdown voltages for a range of gap geometries are summarized in figure 5, taken from Paris & Cortina. For each gap length the absence of polarity effect coincides with a mean stress of 500 to 600 kV/m; a curious feature of the results is that the geometric mean of the positive and negative breakdown voltages for all the gaps is also about 500 to 600 kV/m.

Baldo & Rea (1972) have shown in the case of a vertical rod–rod gap that the axial electric field can be made symmetrical by placing a toroid in a critical position around the live rod; at

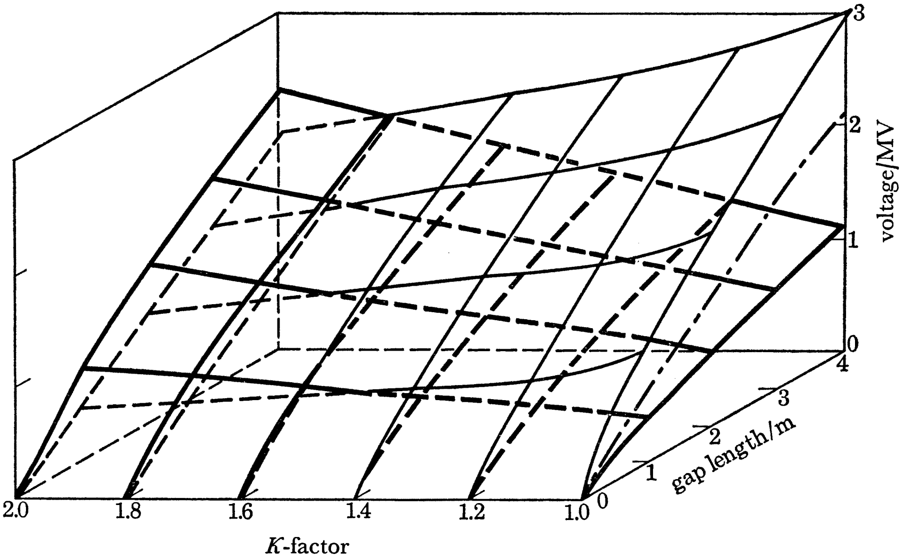


Figure 5. Positive (——) and negative (– – –) breakdown voltage of a range of gaps with different *K*-factor (the ratio of positive switching surge breakdown voltage to that of a rod–plane gap of the same length). Based on Paris & Cortina (1968).

this critical position polarity effect is absent. Similar experiments (H. Duffy, private communication) show that in the critical position, the mean breakdown stress is in excess of 500 kV/m and the dependence on wavefront much less than in the same gap without the toroid.

The effect of field divergence may most easily be studied in sphere–plane gaps for which there is a fair amount of published data. Hepworth, Klewe & Tozer (1972) re-examined the conditions for corona inception on an isolated sphere and calculated the critical electric stress at the surface of the sphere as a decreasing function of radius. Figure 6 shows the ratio of the calculated corona inception voltage to the gap length – 'mean stress' – as a function of gap length and sphere radius. For large spheres and small gaps, standard sphere gap tables show a breakdown voltage

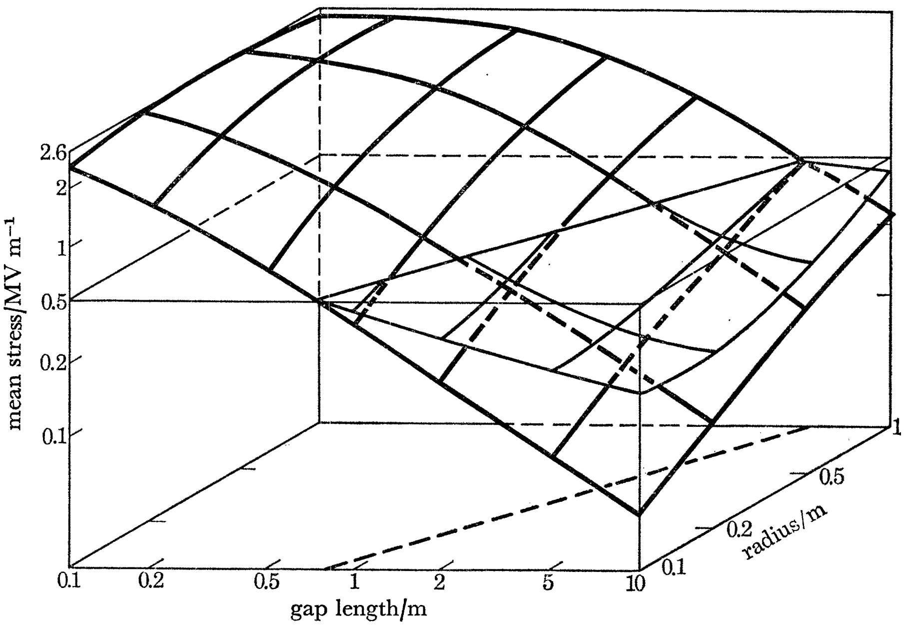


FIGURE 6. Corona inception stress (——) and positive switching surge breakdown voltage (——) of a range of sphere–plane gap lengths and sphere radii. Mean stress is defined as peak applied voltage/gap length. A transition line (——) is shown; to the right of this line corona occurs at a lower voltage than breakdown.

corresponding to a mean stress of around 2.6 MV/m, in agreement with the calculations. For a given sphere radius, as the gap length is increased the measured mean stress for breakdown (Hepworth *et al.* 1972) closely follows the calculated corona inception surface until a discontinuity is reached at a mean stress of about 500 kV/m. The discontinuity also corresponds to a gap length between five and ten times the sphere radius. With further increase in gap length the corona inception stress continues to fall ('Renardières Group' 1972; Ryan & Powell 1972), but the breakdown condition for fast positive impulses remains very nearly constant at ~ 500 to 600 kV/m. For positive switching surges the available data are summarized in figure 6. The minimum stress for breakdown falls as the gap length increases, but less rapidly than the corona inception voltage, becoming less than 200 kV/m for 10 m gaps with small spheres. It can also be seen that, for any given gap length, the switching surge breakdown stress increases significantly with radius, when the radius exceeds about 5 % of the gap length. A practical limit is set, however, by surface contamination of the electrodes in outdoor use. For large spheres, the corona inception voltage is drastically reduced by surface protuberances. It appears (H. W. Whittington, private communication) that when the applied mean stress exceeds 500 to 600 kV/m, the onset of corona is followed immediately by breakdown.

4. Recent experimental evidence on the mechanism of breakdown

In the last year, a considerable amount of new experimental data has become available. Jones & Whittington (1972*b*) applied an optical telemetry technique to the measurement of current leaving the tip of the live electrode in rod–plane gaps of 2 to 10 m, subjected to wavefronts from 10 to 1000 μs. A typical discharge is shown in figure 7, plate 18. The Renardières Group (1972) made comprehensive measurements on rod–plane gaps of 5 and 10 m, with wavefronts from 100 to 500 μs. Particular emphasis was laid on optical measurements by photomultiplier and by image-converters using streak photography and their correlation with voltage and current measurements. The following picture of the breakdown process emerges from these experiments.

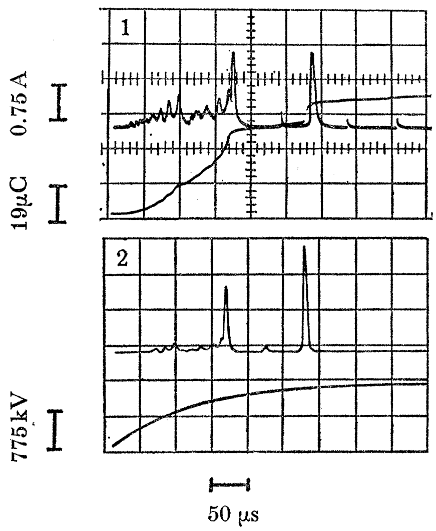
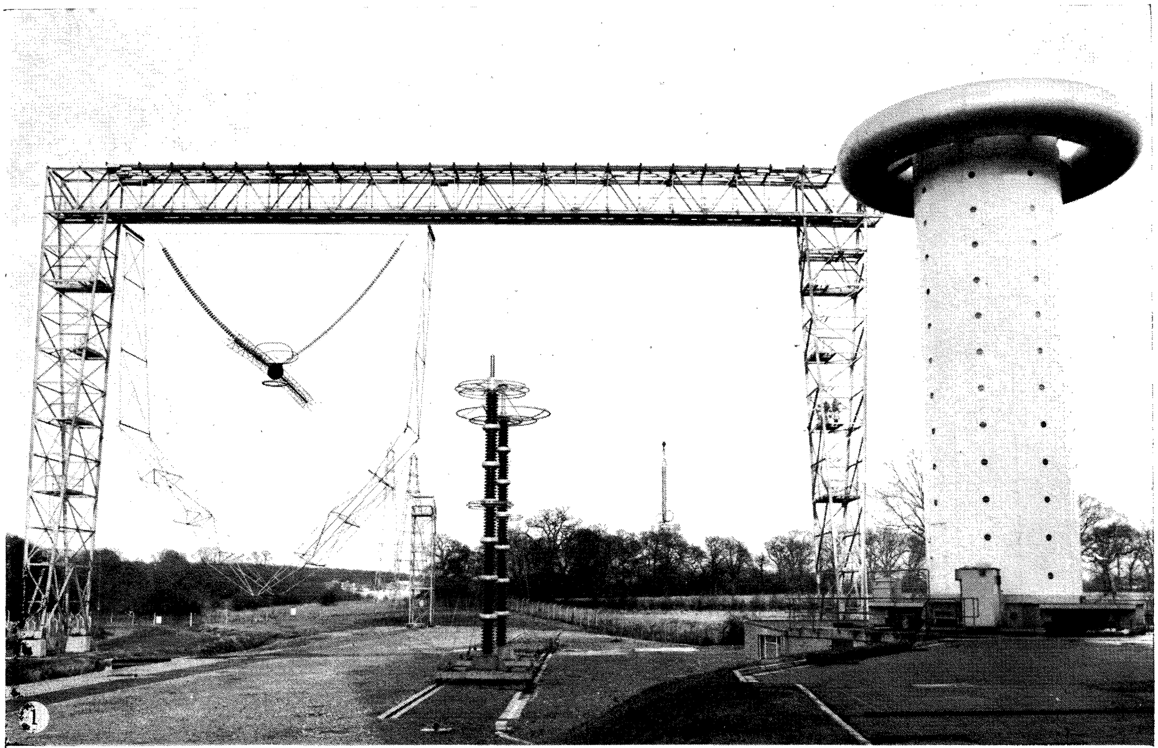


Figure 8. Current, charge, photomultiplier and voltage oscillograms of a single non-breakdown discharge in a 5 m rod–plane gap, at 450 μs time to peak. From the top, current leaving the tip of the rod, total charge injected into the gap, output of photomultiplier (glass optics) viewing an area about 2 m × 2 m including the tip of the rod, voltage across the gap (Jones & Whittington 1972*b*).

As the voltage increases from zero, the onset of corona is detected by the appearance of short (~ 0.1 μs) pulses of current and ultraviolet (u.v.) light. After an interval, when a considerably higher voltage is reached, a luminous charged column develops from the rod electrode. When examined in visible light the column is seen as a narrow (~ 10 mm) thread following an irregular path. In u.v. light, additional diffuse illumination is observed around the tip of the rod and also around the tip of the column; the apparent radius of the column is considerably greater than that seen in visible light. The flow of current from the rod is directly related to the light output (figure 8), and the flow of charge to the distance travelled. Around the critical flashover voltage, the column development may cease part way across when, on average and depending on the time to peak, a charge of between 10 and 30 μC/m of gap length has flowed (figure 9). The least value of charge is found with a time to peak about twice that corresponding to the minimum breakdown voltage; under these conditions the column progresses about 40 % of the distance across the gap.

Three distinct patterns are visible in the pre-breakdown currents, depending on whether the time to peak of the test wave is less than, the same as, or greater than, that value giving minimum breakdown voltage; these will be termed short, medium and long wavefronts respectively.



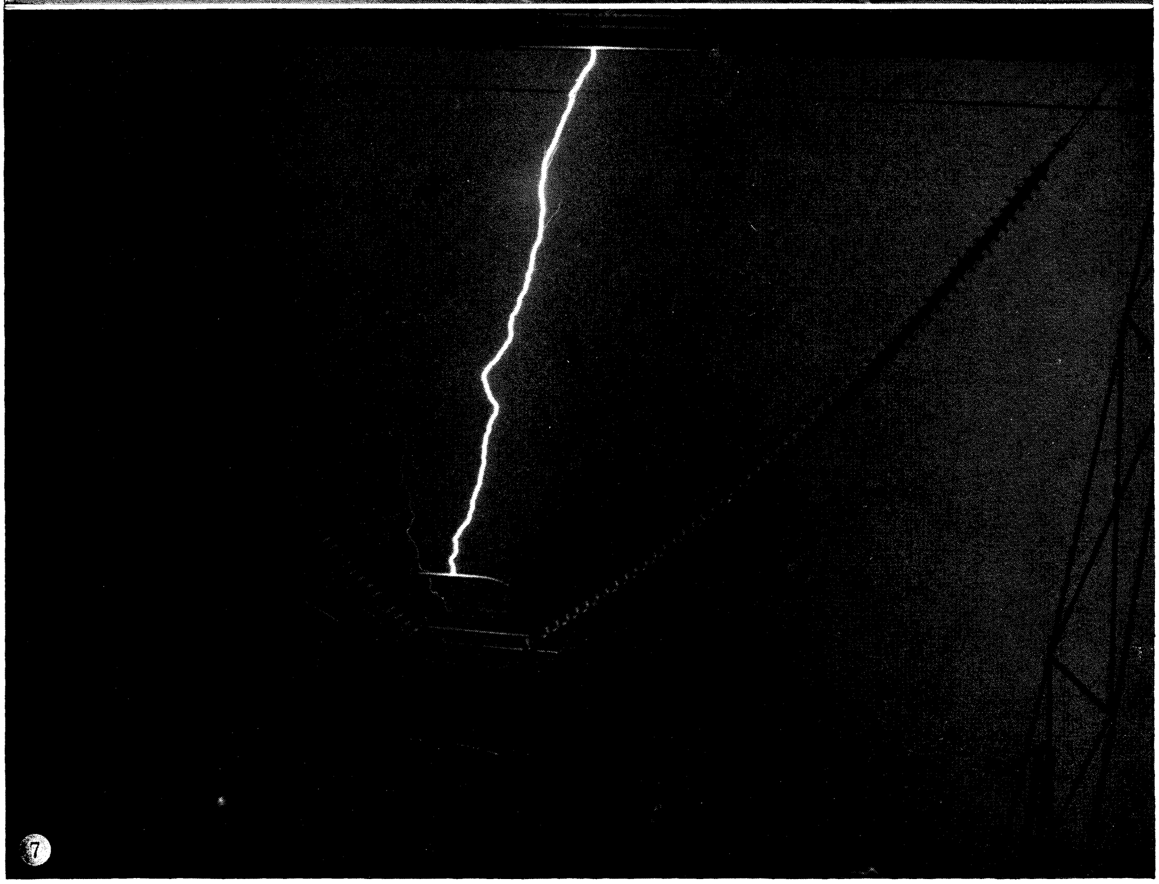


FIGURE 1. General view of u.h.v. laboratory at Central Electricity Research Laboratories, Leatherhead, showing the large outdoor test area, supporting gantry (40 m high by 53 m wide), capacitive voltage divider and load capacitor (centre) and Ferranti all-weather impulse generator, 5.2 MV, 360 kJ (right).

FIGURE 7. Breakdown of a 9.3 m air clearance between simulated 'bundle conductor' and 'window' of a transmission line tower; positive switching surge of 2.2 MV with time to peak around 500 μs. (By courtesy, D. F. Oakeshott.)

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PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

For long-fronted waves, the column advances in steps and several short (10 to 20 μs) pulses of current (a few amperes) and visible light can be detected. Under these conditions, either breakdown occurs or the progress of the column ceases, on the wavefront. For medium-fronted waves, the charge grows almost uniformly with time at a rate corresponding to a current of 0.5 to 1 A, up to a critical point just before the peak of the wave. If breakdown occurs it does so around the peak of the wave; if no breakdown occurs, the current falls to zero at the same point on the wave. The column advances to breakdown at an almost constant average speed of around 20 mm/μs, almost independent of gap length. For short-fronted waves, the column progresses at a steady rate up to the peak of the wave; the current and the rate of progress are almost inversely proportional to the time to peak. After the peak, a relatively smaller current of about 0.5 A flows for a time roughly proportional to the length of the gap. Either the current decays or builds up into a breakdown on the wavetail.

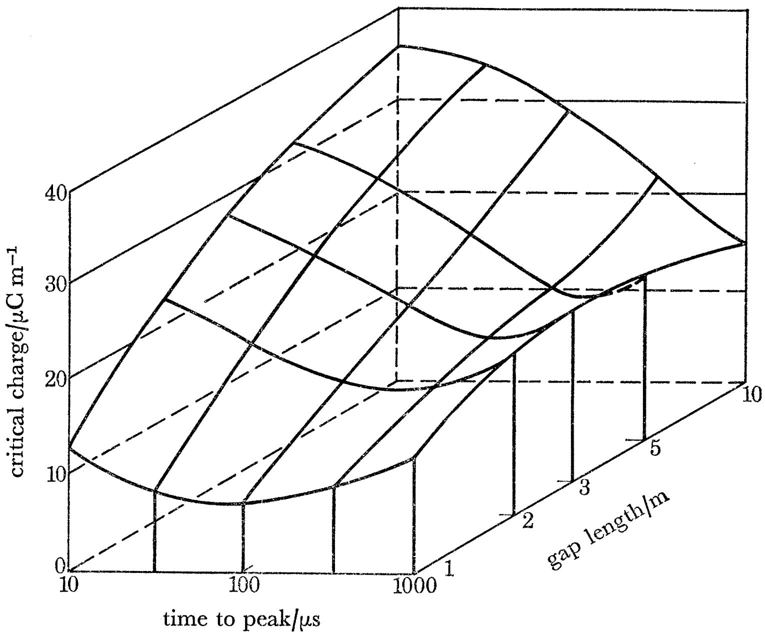

Figure 9. Critical charge (average value for non-breakdown impulse applications at the 50% flashover level) per metre of gap length as a function of gap length and time to peak (Jones & Whittington 1972*b*).

The measurement of pre-breakdown currents has been extended to sphere-plane gaps. For large spheres, the first appearance of corona is immediately followed by breakdown which, at the critical flashover voltage, occurs at the peak of the wave. For small spheres, corona pulses appear early on the wavefront, but the column leaves the electrode only when a higher voltage is reached than that required for corona inception. The addition of a toroid to a rod gap, as described previously, appears not to prevent corona inception but, rather, to delay the start of the propagation of the column.

5. Approaches towards a mathematical model

The review of experimental results has shown that the positive switching surge strength of many air clearances of practical importance can be improved, either by increasing the effective radius of the more highly stressed electrode or by equalizing the stress concentration at the two electrodes. It seems possible to reach, and maintain under contaminated conditions, a mean withstand stress of about 500 kV/m, representing a reduction by a factor of about two in certain of the clearances required for u.h.v. transmission. The problem is to determine the physical conditions for the progress of the breakdown 'column' and the influence of the electrode geometry on these conditions. On the other hand, clearances such as that between an overhead line conductor and earth are not amenable to stress control. The problem here is to assess the sensitivity of the breakdown process to the overvoltage waveforms experienced in service, so as to avoid making tests with every possible waveform. Two simple models have been proposed.

Hepworth *et al.* (1972) proposed that as an increasing voltage is applied to the rod electrode an almost spherical corona cloud grows out from the tip of the rod. At any given voltage the radius of the cloud reaches a stable value at which the electric field at the surface is just insufficient to support the development of further electron avalanches. If the radius of the corona sphere increases to about half the gap length, further expansion can take place with no increase in voltage and breakdown occurs. This criterion explains the measured negative impulse breakdown voltage of sphere–plane gaps. It is found, however, that at the measured positive switching surge breakdown voltage the calculated radius of the corona sphere is only about one-tenth of the length of a rod–plane gap. While the theory accounts well for the observed discontinuity in the positive breakdown stress for sphere–plane gaps (figure 6) it explains neither the observed value of the critical radius of the corona sphere, not the existence of the voltage U-curve.

Bewley (1951) proposed a theoretical criterion, that breakdown of a gap would occur at time t_b if

$$\int_{t_0}^{t_b} (V(t)-V_0)^n \, dt \geqslant k, \quad \text{for} \quad V > V_0, \tag{1}$$

where V_0 was identified with the corona inception voltage, V the instantaneous applied voltage, t_0 the time at which V exceeds V_0, while n and k were arbitrary constants. Taking $n = 2$, Bewley was able to fit the criterion to the measured relation between the peak voltage of short (1/50 μs) impulses and the time to breakdown. In the case of switching surge breakdown, Heilbronner (1972) showed that, for any given gap and wavefront, there is a wide range of combinations of V_0 and n for which k is independent of $V(t)$ and of the time to breakdown. If V_0 is taken to be constant, equation (1) does not predict the rising breakdown voltage for very long wavefronts. The author has found that it is possible to fit the voltage U-curve and to reproduce the observed pattern of front or tail breakdown by making V_0 an arbitrarily increasing function of time.

Although neither of these simple approaches is adequate in itself, the concept of the corona sphere and the need to perform an integration w.r.t. time form the starting-point for the improved model described in the next section.

6. A model of positive switching surge breakdown in rod-plane gaps

Consider a rod–plane gap of length h with an instantaneous voltage V applied to the rod and a column of charge per unit length q and a radius r extending a distance yh from the rod. The condition for further advance depends on the surface gradient and the divergence of the field around the tip; a detailed knowledge of charge density and radius is required for calculation. The need for such knowledge can partially be avoided by the use of a tip potential, P (Davis

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

1963). The value of $P(y)$, which must be equalled or exceeded if further progress is to occur, falls as the tip of the column approaches the plane. The difference between the applied voltage V and the tip potential P is accounted for by a volt drop, ghy, in the column, g being the voltage gradient. For further advance to be possible,

$$V(t) > P(y) + ghy. \tag{2}$$

It is worth noting here that V is a function of time only, determined by the testing circuit; for a particular gap, P is a function of y only, while g depends on the electrical properties of the conducting column. In the following discussion, statistical variability is ignored and the column presumed to pursue a straight course along the axis of the gap.

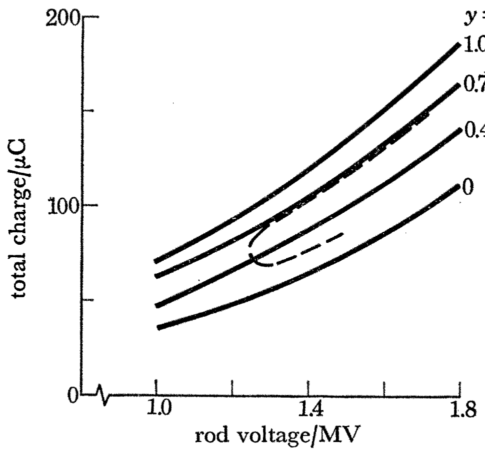

FIGURE 10. Calculated (—) and measured (---) values of charge injected into a 5 m rod–plane gap as a function of the voltage applied to the rod and of the fraction, y, of the gap covered by the column. Measured values are plotted as the locus of critical charge (figure 8) and 50 % flashover voltage (figure 3) for a range to time to peak from 10 to 1000 μs; the upper branch corresponds to short times to peak.

In considering the function $P(y)$ it is necessary to examine the distribution of charge. The total charge, Q, injected into the gap is the sum of Q_s, that of the corona sphere, and qhy, that of the column. For any given radius of the column, a particular value of charge density (presumed uniform) is required to maintain the tip potential. On the other hand, only certain combinations of charge density and radius will maintain the radial field strength at the critical value for corona inception (~ 3 MV/m). In the case of the rod–plane gap, quite simple approximations may be derived (appendix). Figure 10 shows the total charge in a 5 m rod–plane gap as a function of the applied voltage and the fractional distance, y, reached by the column. Also shown in figure 9 is the locus of average measured values of charge leaving the tip of the rod, in those 50 % of cases where breakdown did not occur, and the critical flashover voltage, over a range of wavefronts (figures 9 and 3). Similarly, agreement is obtained with the measured data for 2, 3 and 10 m gaps. It appears that the fractional distance, y, at which the progress of the column ceases ranges from ~ 0.7 with short wavefronts to ~ 0.2 with long wavefronts; the value of $y = 0.4$, deduced for wavefronts giving maximum breakdown voltage, is in fair agreement with direct optical evidence (Renardières Group 1972).

The conditions for the transition from corona sphere to column at the start of the breakdown process may now be considered. In the appendix, expressions are derived for the stable radius of a column as a function of tip potential, in a rod–plane gap and also for the length of an electron avalanche at the surface of a corona sphere. The stable radius of the column increases more rapidly with tip potential than does the avalanche length associated with a corona sphere at the same potential. At lower voltages, the avalanches are longer than the radius of a prospective column and it seems unlikely that such a column can form. At higher voltages it seems that provided further avalanches can start from the tip, the development of a column rather than further growth of the sphere should be favoured. If a theoretical criterion is assumed, by equating the avalanche length to 1.5 times the radius of the column, the voltage, P_0, at which progress of the column starts is found to be

$$P_0 \sim 0.27 h^{0.5} \text{ (MV, m)}.$$

The starting voltage for column propagation can be derived from current or photomultiplier oscillograms. A regression analysis of measured average values in gaps of 2 to 10 m leads to an approximate expression

$$P_0 \sim 0.27\, h^{0.55} \text{ (MV, m)}.$$

Suppose the column to stop part way across the gap, and the tip potential to rise slowly, following the increase in the applied voltage. A charge sphere grows from the tip until the stable radius exceeds the critical value and a further length of column propagates. A useful property of the rod–plane geometry is that the field pattern around the tip is unlikely to be greatly altered if the column is replaced by an extension of the rod. In other words, the critical tip potential $P(y)$ approximates to the column starting voltage in a gap of length $h(1-y)$. The mathematical model of column progress smooths out any fluctuations and assumes a steadily advancing column. For rod–plane gaps, using the previously derived expression,

$$P(y) = 0.27 h^{\frac{1}{2}} (1-y)^{\frac{1}{2}} \quad \text{(MV, m)}.$$

For ideal rod–rod gaps, consideration of the symmetry of the field suggests that the constant should be higher by a factor $\sqrt{2}$, or 0.38 MV. For other electrode geometries, more elaborate calculations are required. The important point established is the possibility of approximate representation of the complex charge distribution and corona inception conditions by a single function $P(y)$, defined along the path followed by the column.

Using the values of P derived above, equation (2) shows that the breakdown voltage is always greater than gh. The equation predicts that the mean stress for breakdown will be higher for rod–rod than for rod–plane gaps and higher for shorter gaps. Measured critical flashover voltages (Bowdler & Hughes 1960) with positive 1/50 μs impulses, for which breakdown occurs in a few microseconds, are consistent with a value for g of 500 kV/m. It seems that the voltage U-curve can only be explained if the column gradient varies with the rate of progress of the column.

The rate of growth of the column can be found from differentiation of equation (2) with respect to time,

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\mathrm{d}P}{\mathrm{d}y}\frac{\mathrm{d}y}{\mathrm{d}t} + hg\frac{\mathrm{d}y}{\mathrm{d}t} + hy\frac{\mathrm{d}g}{\mathrm{d}t}, \tag{3}$$

or

$$y' = V'h - yg' \Big/ g + \frac{1}{h}\frac{\mathrm{d}P}{\mathrm{d}y} \tag{4}$$

$(\mathrm{d}P/\mathrm{d}y)$ is always negative and its magnitude increases for values of y approaching unity. A point is reached in the progress of the column where the denominator reaches zero and breakdown follows in a sudden jump. In rod–plane gaps, the values are such that about the last 10 % of the gap is traversed in a sudden jump. If the configuration of the electrodes is such that $|1/h\, \mathrm{d}P/\mathrm{d}y|$ approaches 500 kV/m for all values of y, breakdown occurs very quickly.

If g is constant, $g' = 0$ and the breakdown voltage is independent of the wavefront duration. In the marginal case, breakdown occurs at the peak of the wave; with increase of applied voltage breakdown occurs earlier on the wavefront, at the same instantaneous voltage. If g' is negative, the column continues to progress on the tail of the wave, until the negative value of V' becomes too great. If g' is positive, the progress of the column ceases before the peak of the wave.

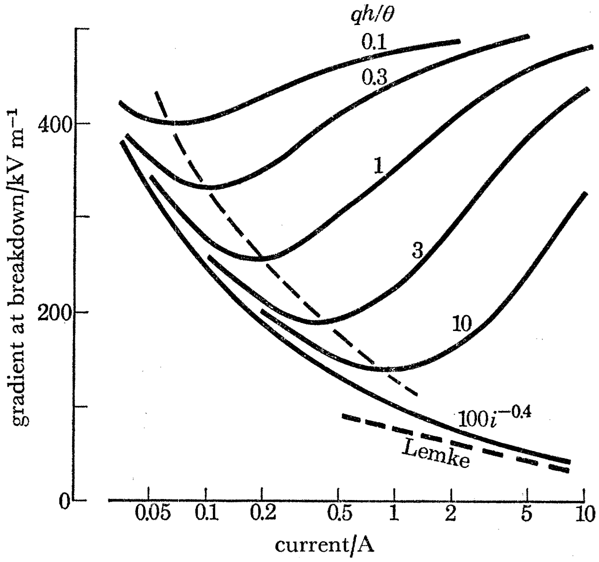

FIGURE 11. Gradient of column at breakdown, evaluated from equation (7), as a function of average current flowing during the breakdown process. The parameter qh/θ is charge per unit length of column × gap length ÷ effective time constant of the column. $100i^{-0.4}$ is the assumed static characteristic; — — static characteristic given by Lemke (1968).

By analogy with the switchgear arc, which has been studied extensively the column gradient may be regarded as having a static characteristic $g_0(i)$ – an inverse function of current – and a time constant θ. The current in the column is given by

$$i = \frac{\mathrm{d}(qhy)}{\mathrm{d}t} = qhy' + hy\,\frac{\partial q}{\partial V}\,V'. \tag{5}$$

The first term corresponds to the addition of new charge as the column progresses, the second term to the displacement current flowing as the potential of the whole column is raised. In the appendix, an approximate expression is derived for the gradient g at time t on the assumption of a constant current, corresponding to conditions around the minimum of the voltage U-curve,

$$g = g_0 + (g_i - g_0)\exp(-t/\theta'), \tag{6}$$

where θ' is an effective time constant (not necessarily with any direct physical significance) and g_i the initial gradient at time zero. The instantaneous value of g varies along the length of the column, since the parts nearest the electrode have been in existence for a longer time. The total volt-drop should, in principle, be found from integration over the existing length. In the special case of a column progressing at a uniform speed, it is sufficient to take an average over time. Breakdown will occur at a time $t_b = qh/i$ and the corresponding average gradient, g_b, will be given by substituting this expression for t_b as,

$$g_b = g_0(i) + (g_i - g_0(i))\,\frac{\theta_i'}{qh}\,(1 - \exp(-qh/\theta_i')). \tag{7}$$

The column gradient at breakdown, g_b, is shown in figure 11 as a function of current i, with (qh/θ') as parameter. The minimum value of g_b falls uniformly with (qh/θ') and occurs with current in the range 0.1 to 0.3 A.

It is clear that the behaviour of g_b bears a close resemblance to the voltage U-curve. The constant and exponent in the column gradient characteristic and the effective time constant can at this stage only be obtained by comparison with measurements of the minimum breakdown voltage and corresponding current over a range of gap lengths. By trial and error it is found that only certain combinations of the column parameters lead to good agreement with measurement; it seems possible to isolate each parameter within a range of about $\pm 30\,\%$. A good fit can be obtained with $g_0 \sim 100i^{-0.4}$ (kV, m, A) and $\theta' \sim 50$ µs. The static characteristic derived in this way corresponds fairly well to direct measurements over the range 1 to 10 A (Lemke 1968). So far as is known, there is no published data appropriate to the region of interest – 0.1 to 1 A.

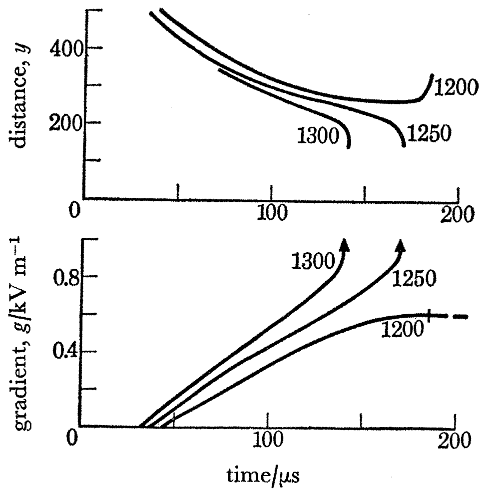

FIGURE 12. Calculated gradient and distance travelled by the column as a function of time for a 5 m rod–plane gap subjected to an exponential wavefront of time constant 50 µs, with applied peak voltages of 1200, 1250 and 1300 kV.

The effect of changes in waveshape may be investigated by the simultaneous solution of the differential equations for y' and g' (equations (4) and (A 9)), starting at the time at which the applied voltage exceeds the column starting voltage. A typical solution (obtained by standard techniques of numerical analysis) is shown in figure 12 for a 5 m rod–plane gap, subjected to an exponential wavefront with a time constant of 50 µs (approximately 200 µs to peak). The solutions show that at a low applied voltage the gradient falls initially and then rises, while the progress of the column ceases part way across the gap. At a higher applied voltage, the gradient falls continuously; the speed of progress is fairly constant until most of the gap has been traversed, when breakdown occurs in a final jump. Similar calculations for longer or shorter wavefronts lead to a higher breakdown voltage and show that breakdown occurs or the progress of the column ceases on the wavefront or wavetail respectively. In the non-breakdown cases, the distance reached by the column is sensitive to the peak applied voltage. A similar sensitivity is found to variations in the time constant or static characteristic of the column; the statistical variability of these quantities leads to the random occurrence of and variable times to break down, at constant applied voltage.

Extension of the calculation to different simple waveshapes, e.g. (1-cosine) or linear ramp, predicts significance differences (5 to 10% for 5 m rod–plane gaps) in the minimum breakdown voltage and large differences in the time to peak at which the minimum occurs.

The model seems to be directly usable for the interpolation and limited extrapolation of existing test data. More complex calculations are required for the critical tip potential in gaps other than rod–plane; in particular the criterion for growth of a column, rather than further expansion of a corona sphere, needs closer examination. Finally, an improved physical understanding of the volt/ampere characteristics of the column (or leader) is required.

7. Conclusions

1. The switching surge withstand strength of many practical air clearances and the performance of protective gaps may be materially improved by modification of the electric field around the electrodes; a practical upper limit is ~ 500 kV/m for outdoor insulation.

2. A model is proposed which seems capable of simulating all the important features of the switching surge breakdown process; fair quantitative agreement has been obtained in the case of rod–plane gaps. Significant differences in withstand voltage are predicted, depending on the shape as well as the duration of the applied surge.

This work was done at the Central Electricity Research Laboratories and is published by permission of the Central Electricity Generating Board.

Appendix

The field, E, at the surface of an isolated sphere, radius r with a charge Q is given by $E = Q/4\pi\epsilon_0 r^2$. If a corona cloud of radius r is not to grow further it may be deduced from the results of Hepworth *et al.* (1972) that the surface field should be less than about $2.8r^{-0.15}$ (MV, m) Remembering that the potential $V = Q/4\pi\epsilon_0 r$ and inserting the limiting field conditions it can be shown that, approximately,

$$r_s \sim 0.3\,V^{1.15}\ \text{(m, MV)} \quad \text{and} \quad Q_s \sim 34V^{2.15}\ (\mu\text{C, MV})$$

where r_s may be regarded as the stable radius associated with a given potential. Hepworth *et al.* also calculated the electron avalanche length, l, as a function of sphere radius at the corona inception voltage. From their results, substituting voltage for radius,

$$l \sim 0.05V^{0.75} \quad \text{(m, MV)}.$$

Davis (1963) showed that the potential V_t at the tip of a vertical column of radius r and charge q per unit length is given by

$$V_t = \frac{q}{2\pi\epsilon_0}\left(\ln\frac{4h}{r} - \frac{2h}{H}\right), \tag{A 1}$$

where the column extends vertically from h to H above the earthed plane. The radial field at the surface of the column is given by $E = q/2\pi\epsilon_0 r$. Further radial corona growth is presumed to be limited as in the case of the sphere described above. The tip potential may then be written as

$$V_t = \frac{q}{2\pi\epsilon_0}\left(\ln 4h - 1.17\ln(q/2.8\times 2\pi\epsilon_0) - 2h/H\right). \tag{A 2}$$

The charge per unit length of the column, q, may be plotted as a function of the tip potential for several values of h, with $H = 2h$. The stable radius, r, of the column may be derived by simple substitution and also expressed as a function of the tip potential. Over the range of interest, approximate expressions may be derived graphically

$$q \sim 18V^{1.3}h^{-0.3} \quad (\mu\text{C, MV, m}),$$

$$r \sim 0.09V^{1.52}h^{-0.37} \quad (\text{m, MV}).$$

To allow for the potential gradient in the column, it seems reasonable to estimate an average value of q at a potential midway between that of the rod and the tip of the column; this has been done in the derivation of figure 10. The estimates of q obtained in this way are in the range 10 to 20 μC/m, and agree well with more precise calculations made for specific cases by Baldo, Gallimberti, Rea & Zingales (1972).

The current at the start of column propagation, i_0, may be estimated by setting $y = 0$ in equations (4) and (5) as

$$i_0 = qhy' = qV' \Big/ \left\{g + \frac{1}{h}\frac{\partial P}{\partial y}\right\}. \tag{A 3}$$

Evaluating this expression, with $g = 500$ kV/m and values of charge, q, appropriate to the starting voltage ($V_s = 0.27h^{0.5}$) gives, approximately,

$$i_0 \sim 8h^{0.45}V' \quad (\text{A, m, MV, }\mu\text{s}).$$

The current flowing from the tip of the rod contains an additional term, $\partial Q_s/\partial V\, V$ due to the corona sphere which, similarly, can be shown to be

$$i_s \sim 16h^{0.55}V'.$$

A regression analysis of measurements (Jones & Whittington 1972*b*) of the average initial rate of charge growth in rod–plane gaps of 2 to 10 m, with wavefronts ranging from 7 to 700 μs gives

$$i_{\text{meas}} \sim 25h^{0.6}\,\hat{V}/t_f,$$

where V' is approximated by the ratio of peak voltage, $\hat{V}$ to virtual front duration, t_f. When the small errors in representing V' by $\hat{V}/t_f$ are taken into account, the agreement is satisfactory. It should be noted that only about one-third of the current leaving the tip of the rod is associated with the growth of the column.

Following Grütz, Hochrainer, Schwarz & Thiel (1972) a dynamic equation for the conductivity, $\sigma = g/i$, may be written as

$$\frac{d\sigma}{dt} = \frac{1}{\theta}(\sigma_0 - \sigma),$$

or

$$\frac{dg}{dt} = \frac{g^2}{\theta}\left(\frac{1}{g} - \frac{1}{g_0}\right) + \frac{g}{i}\frac{di}{dt}, \tag{A 4}$$

where $g_0(i)$ is the static characteristic and θ the time constant.

Consider the effect of the application of a series of pulses of current of amplitude i and duration τ occurring at intervals of T. During the interval (τ) the change in gradient, δg, is given by

$$\delta g = \frac{\tau g}{\theta}\left(1 - \frac{g}{g_0}\right) + (F),$$

and during the interval $(T-\tau)$ by

$$\delta g = \frac{(T-\tau)g}{\theta}\left(1 - \frac{g}{g_i}\right) - (F),$$

where (F) is a transient associated with the $(\mathrm{d}i/\mathrm{d}t)$ term in equation (A 4), g_0 is the steady-state gradient appropriate to the current i and g_i that corresponding to zero current $(g_0 < g < g_i)$.

The total change over the interval T is the sum of these two changes,

$$g = \frac{Tg}{\theta} - \frac{g^2}{\theta}\left\{\frac{\tau}{g_0} + \frac{T-\tau}{g_i}\right\}. \quad \text{(A 5)}$$

If $\tau \to T$, approximating to a constant current,

$$\delta g = \frac{Tg}{\theta}\,(1 - g/g_0)$$

and g falls continuously to g_0.

If $\tau \to 0$,

$$\delta g = \frac{Tg}{\theta}\,(1 - g/g_i)$$

and g rises continuously to g_i. Using typical values of $g_i = 500$, $g = 300$, $g_0 = 100$ kV/m it can easily be shown that for $\delta g \sim 0$, $\tau \sim T/6$. It appears that current 'spikes' of short duration relative to the pulse interval are associated with a rising gradient: almost continuous current with a falling gradient. For the purpose of further calculation, it also seems desirable and probably valid to insert in equation (A 4) a value of g_0 appropriate to the average current and to neglect the term in $(\mathrm{d}i/\mathrm{d}t)$.

The solution of equation (A 4) then becomes

$$g = \frac{g_i g_0}{g_i - (g_i - g_0)\exp(-t/\theta)}, \quad \text{(A 6)}$$

where g_i is the initial gradient at $t = 0$. Since, however, it is necessary to perform an integration over distance, or time, to obtain the average gradient of the complete column, an even simpler equation is desirable:

$$g' = (g_0 - g)/\theta'. \quad \text{(A 7)}$$

The solution is $g = g_0 + (g_i - g_0)\exp(-t/\theta')$ and, over the region of interest, can be shown to be a good approximation to equation (A 6) if $\theta' \sim \frac{1}{2}\theta$. The average value of g along the length of a column progressing at a constant speed may be found from the time average of equation

$$g_a = g_0 + (g_i - g_0)\,(\theta'/t)\,(1 - \exp(-t/\theta')). \quad \text{(A 8)}$$

It can be shown that the term $\theta'(1 - \exp(-t/\theta'))/t$ may be approximated by $\exp(-t/\theta'')$ where $\theta'' \sim 2\theta'$. The final simplified differential equation then becomes

$$g' = (g_0 - g)/\theta''. \quad \text{(A 9)}$$

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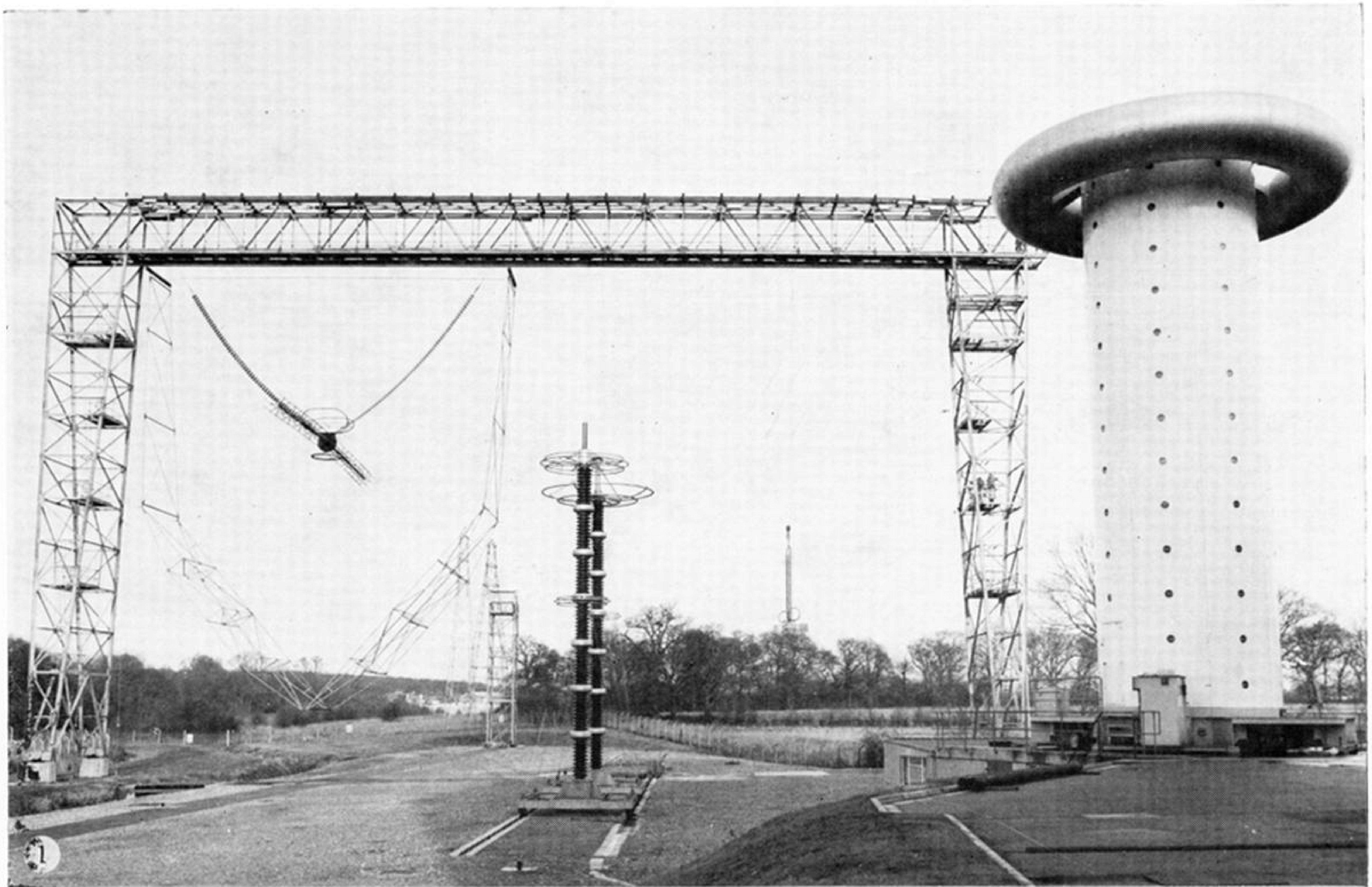

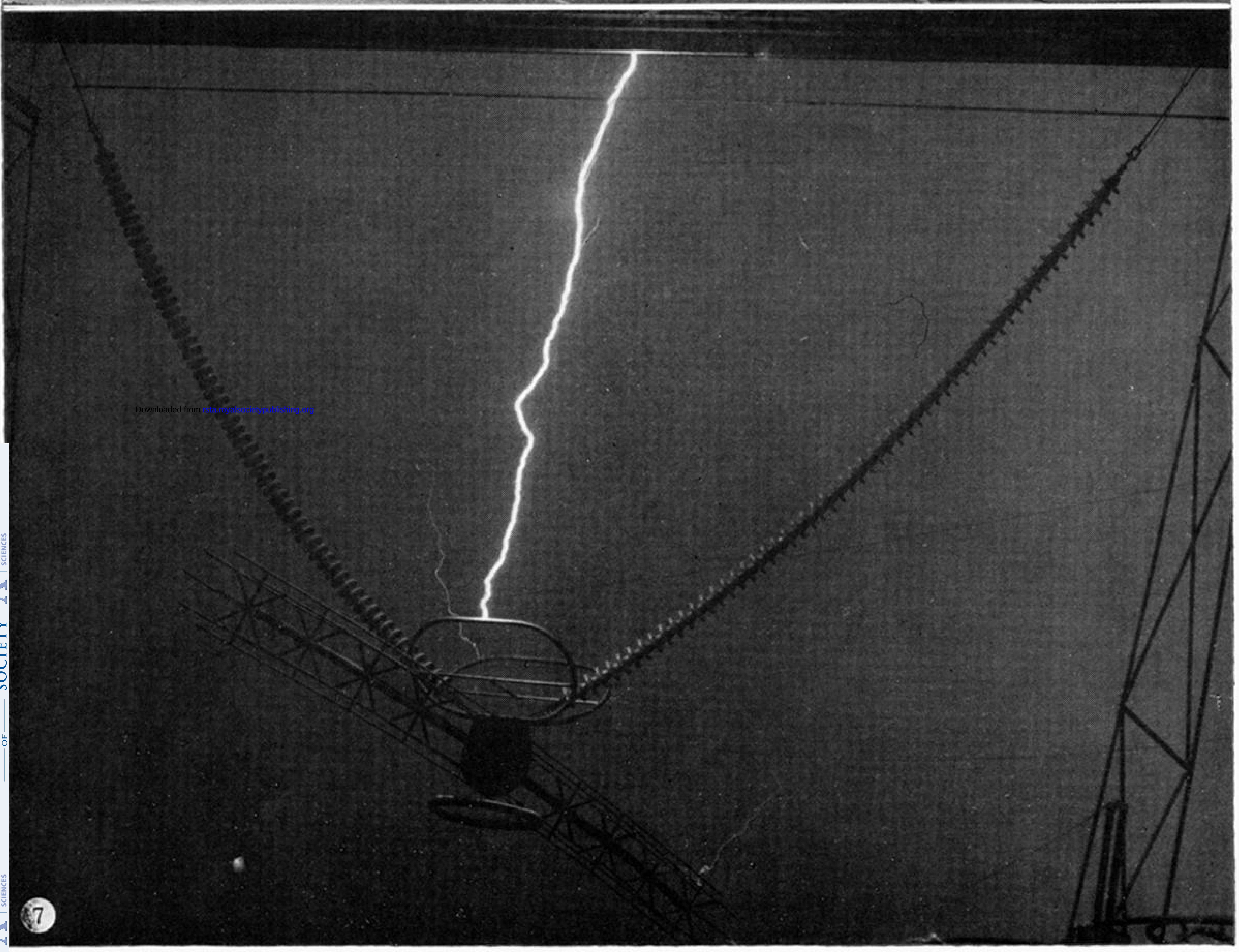

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

FIGURE 1. General view of u.h.v. laboratory at Central Electricity Research Laboratories, Leatherhead, showing the large outdoor test area, supporting gantry (40 m high by 53 m wide), capacitive voltage divider and load capacitor (centre) and Ferranti all-weather impulse generator, 5.2 MV, 360 kJ (right).

FIGURE 7. Breakdown of a 9.3 m air clearance between simulated 'bundle conductor' and 'window' of a transmission line tower; positive switching surge of 2.2 MV with time to peak around 500 μs. (By courtesy, D. F. Oakeshott.)